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Simple, complete, and novel quantitative model of holography for students of science and science education

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Simple, complete, and novel quantitative model of holography for students of science and science education

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Abstract. A Moiré graphical method for predicting the directions of all permitted image waves for thin and volume holograms is described. This method has been developed in connection with a holography centred optics course for students in science, technology and science education that has evolved over several decades. A somewhat unique view of the holographic process underlies this novel method of predicting the behaviour of diffraction gratings.

1. Moiré models for wave interference reviewed: Hologram Formation

Moiré plane wave models, like those shown in figure 1, are widely used in introductory books on holography to represent wave interference and hologram formation. A comparison of figures 1a and 1b demonstrates that as the angle of the object beam relative to the reference beam *increases* in magnitude, the bright fringe spacing recorded in a thin or volume transmission hologram *decreases* in magnitude. This semi-quantitative demonstration illustrates the usefulness of Moiré diagrams. In this paper the application of Moiré diagrams will be extended to include prediction of permitted image wave directions for thin and volume holographic gratings.

Moiré plane wave diagrams not only provide a clear visual representation of the hologram recording process, but also yield geometric diagrams that lead to main holographic equations. For example, the well-known triangle diagram shown in figure 2 leads to the *light wave fringe spacing equation*, equation 1. All equation 1 symbols are explained in the caption for figure 2.

$$\Lambda_{\text{JW}} = \lambda/[2\sin \theta^*] \quad (1)$$

Equation 1 is obtained by utilizing right triangle HGB in figure 2.

Using the symmetry between the two interfering plane waves shown in figure 2, we can know that the object and reference beam direction vectors make equal and opposite angles, θ^* , and $-\theta^*$, with the bright fringe direction. Alternately stated, the bright light fringes bisect the angle $2\theta^*$ between the two wave directions. We might call this the *fringe direction rule*.

To relate the light wave interference diagram of figure 2 to experimental arrangements involving holographic film, we can introduce angles α_O and α_R which, respectively, tell the directions of the

object and reference beams relative to the film normal. The two angles α_O and α_R are shown in figure 3.

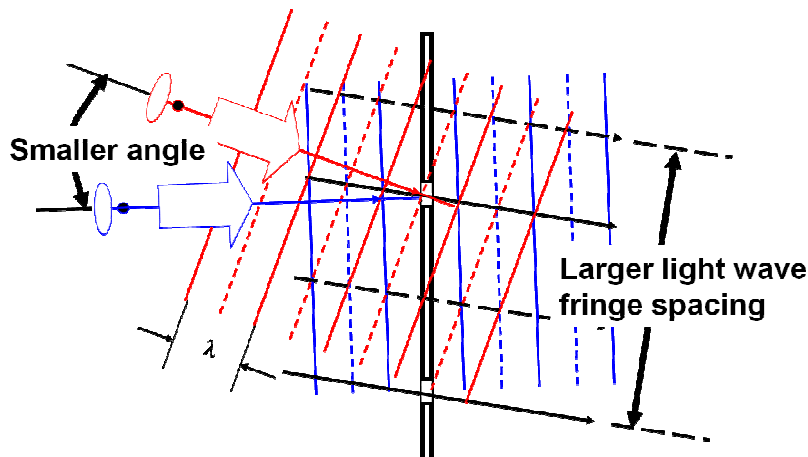


Figure 1a. Moiré pattern represents hologram formation. The reference beam angle of incidence, not marked in the film, is -20° (CW). The object beam angle of incidence is $+2^\circ$, both relative to film normal.

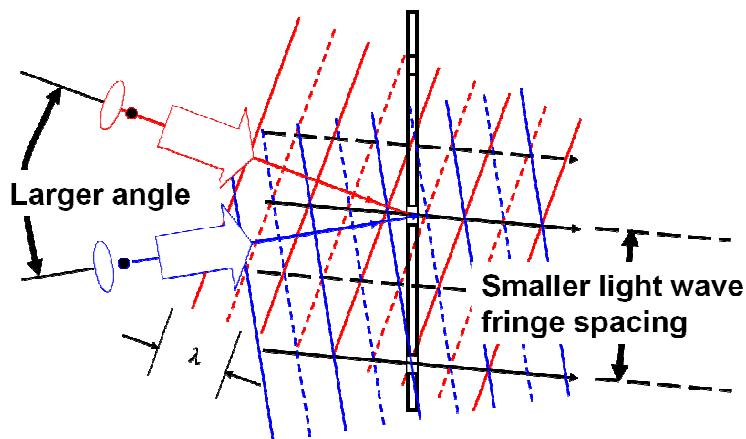


Figure 1b. Compared with Fig. 1a, the object beam angle of incidence is increased to $+10^\circ$, while reference beam remains unchanged in direction at -20° .

Using the object beam and direction beam direction angles α_O and α_R shown in figure 3, we can then express the light wave fringe direction (ϕ) relative to the film normal by the *light wave fringe direction equation*, equation 2.

$$\phi = [\alpha_O + \alpha_R]/2 \quad (2)$$

Also it is helpful to write equation 3 which defines θ^* in terms of α_O and ϕ .

$$\theta^* = \alpha_O - \phi \quad (3)$$

Then, combining equations 2 and 3, we obtain a second equation for θ^* .

$$\theta^* = [\alpha_O - \alpha_R]/2 \quad (4)$$

The inverse equations are shown as equation 5 and equation 6.

$$\alpha_O = \phi + \theta^* \quad (5)$$

$$\alpha_R = \varphi - \theta^* \quad (6)$$

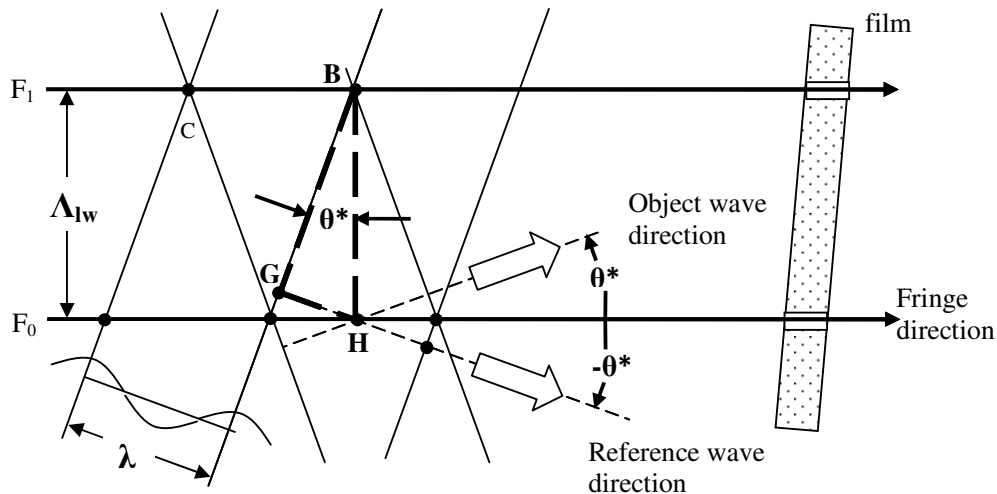


Figure 2. Figure 2 utilizes right triangle HGB to establish the relationship between light wave fringe spacing ($BH = \Lambda_{LW}$), light wavelength ($GH = \lambda/2$), and the angle ($\theta^* = \angle GBH$). θ^* is the angle by which the object wave direction is rotated out of the light wave fringe direction. We take CCW rotations as positive. Directed lines F_0 and F_1 represent bright fringes carrying wave energy. Angle θ^* relates the object and reference beam directions to the light wave fringe directions, but contain no information about wave or fringe directions relative to the film.

Equations 1 through 6 lead rather directly to two useful equations for the hologram fringe spacing (Λ_S) along the surface of a hologram. The *film surface fringe spacing equation*, version one is shown in equation 7.

$$\Lambda_S = \Lambda_{LW} / \cos \varphi \quad (7A)$$

$$\Lambda_S = \lambda / (2 \sin \theta^* \cos \varphi) \quad (7B)$$

An equivalent equation, but with a form that is more closely related to the thin grating equation used to describe hologram viewing is shown in equation 8. Again, equations 7 and 8 are mathematically equivalent.

$$\Lambda_S = \lambda / (\sin \alpha_O - \sin \alpha_R) \quad (7B)$$

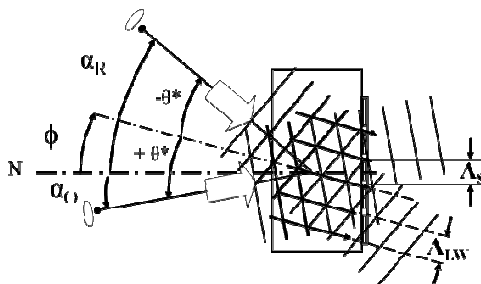


Figure 3. All important angles and fringe spacings associated with hologram formation are shown. The angles are discussed, and equations related the various angles are provided in the text.

Λ_S is the film surface fringe spacing equation, often denoted elsewhere as d , for the case of a thin grating.

Λ_{LW} is the light wave fringe spacing.

2. A novel Moiré graphical method for predicting holographic grating image wave direction

Figure 4 introduces a Moiré graphical method for locating all permitted image waves for a thin holographic grating with wavelength to grating space ratio equal to 0.5. The viewing wave direction is $\beta_v = -20^\circ$ throughout. The first and second order image directions are found using the *method of hypothetical fringes*, the fringes that would be formed by the interference of the viewing wave and the proposed image wave, should they be permitted to pass through the same region of space.

A complete set of equations for implementing the hypothetical fringe model more efficiently has been written out by the author, and will be presented elsewhere. A main point is that, because the hypothetical fringe method casts predictions about hologram image waves, i.e. hologram viewing into the ‘same mold’ as that used for hologram formation, all hologram viewing equations turn out to be essentially identical to those that describe hologram formation. The hologram viewing equations then become relatively easy to write down ‘from scratch’ and without reference to traditional diffraction grating theory. The logic underlying hologram imaging and hologram formation become essentially the same when the hypothetical fringe method is utilized. All derivations are based on the same type of Moiré diagrams, so the similarity between the hologram formation and the hologram viewing equations becomes less surprising. In particular it should be noted that the diagrams that quantitatively model hologram formation, shown in figures 1, 2 and 3 are essentially identical to those that model thin hologram imaging, shown in figure 4. Then, though volume hologram imaging has not been treated in similar detail in this essay, the explanation in figure 5 of the well known ‘red shift’ seen as a white light viewing beam is rotated nearer to the normal of a white-light viewable reflection indicates, correctly, that the hypothetical fringe model works equally well for thin and volume holographic gratings.

3. Discussion

In the opinion of this author a problem with traditional approaches to modeling holography is that hologram formation and hologram viewing appear to be ‘disparate processes.’ The fact that the hologram will form an image wave that necessarily is identical, in many ways, to the object wave used to form the hologram does not follow directly from the model or concepts introduced. Using thin amplitude transmission holograms as an example, in the traditional models, the stable wave interference pattern formed by the object and reference beam is shown to cause a pattern of microscopic clear and dark regions, called fringes, to be recorded in a light sensitive medium, such as silver halide film. Then, the chemically process film, the hologram, is interpreted as being a grating much like a traditional diffraction grating, and, correctly, traditional diffracting grating principles are applied. Again, the fact that the object wave will be spontaneously reconstructed if the hologram is correctly illuminated doesn’t seem natural and obvious.

This essay has demonstrated a method that the author has not seen implemented elsewhere, and which has the potential to better integrate the two parts of holography, hologram formation and hologram viewing. The conceptual reasoning used by the author to ‘discover’ this method will be outlined briefly here. The main point is to establish a viewpoint such that the ‘magic’ of the holographic process will appear as a natural outcome of wave interference principles. The goal is to retain the magic, while removing the mystery

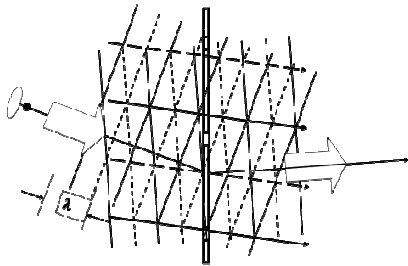


Figure 4a. Trial image wave direction $\beta_{+1} = 4^\circ$.

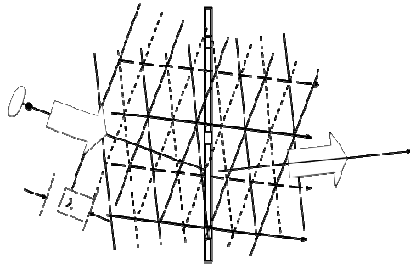


Figure 4b. Trial image wave direction $\beta_{+1} = 6^\circ$.

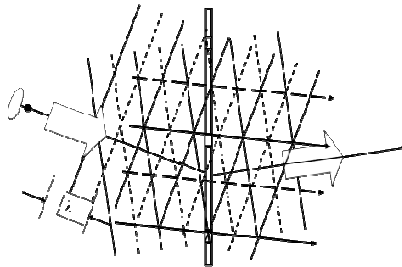


Figure 4c. Trial image wave direction $\beta_{+1} = 8^\circ$.

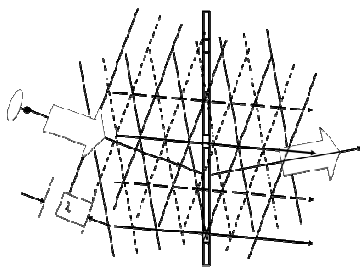


Figure 4d. Trial image wave direction $\beta_{+1} = 10^\circ$.

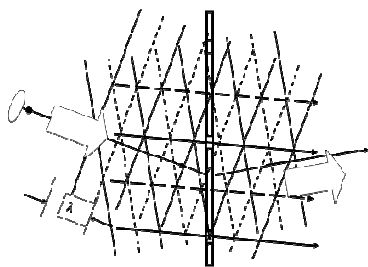


Figure 4f. $\beta_{+1} = 9^\circ$. This is the correct direction!

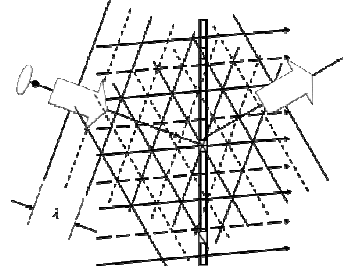


Figure 4g. Trial image wave, $\beta_{+2} = 30^\circ$.

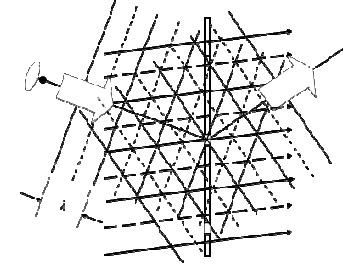


Figure 4h. Trial image wave, $\beta_{+2} = 34^\circ$.

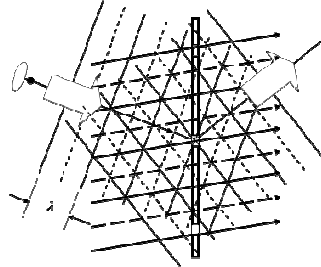


Figure 4i. Trial image wave, $\beta_{+2} = 38^\circ$.

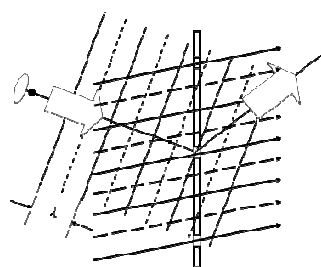
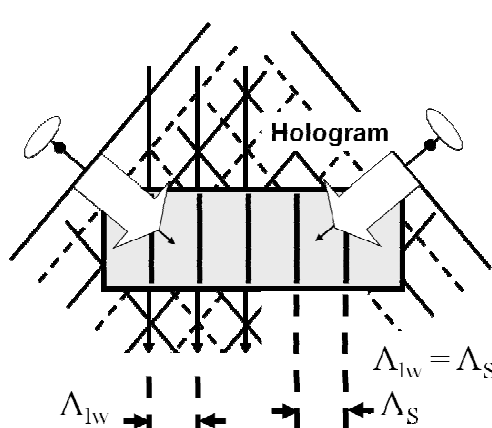
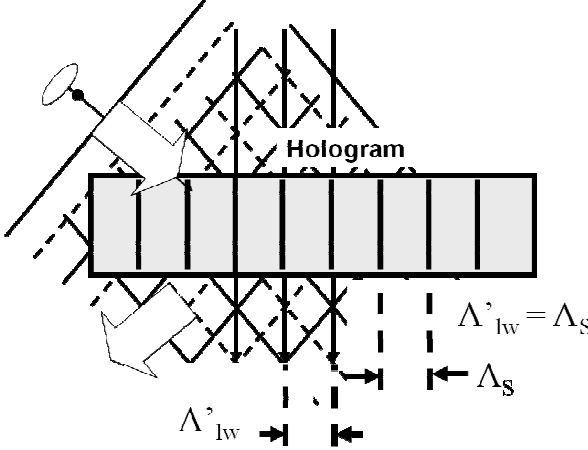
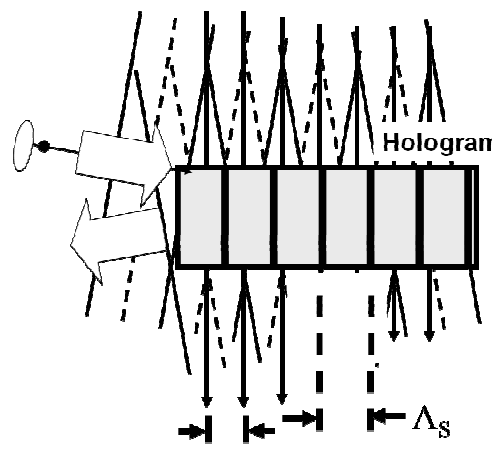


Figure 4j. Trial image wave, $\beta_{+2} = 41^\circ$, shown immediately above. This is the correct direction for the second order ($=2$) image wave. A bright 'hypothetical fringe' passes through every grating opening, so this is an acceptable image wave. There are two bright fringes for every grating opening, making this the second order ($m = +2$) image wave. Figure 4f shows the correct first-order ($m = +1$) image wave at $\beta_{+1} = 9^\circ$.

Figure 5 applies the method of hypothetical fringes to describe the red shift that occurs for a volume hologram, as the viewing beam is brought closer to normal incidence.

	<p>Figure 5a. The formation of a reflection hologram with fringes parallel to the film surface is shown at the left.</p> <p>For simplicity we assume the index of refraction of the film is $n = 1$, so the light wave fringe structure, and the fringe structure recorded in the hologram</p> <p>The fringe spacing along the film surface and in the film will be equal to the light wave fringe spacing. $\Lambda_s = \Lambda_{LW}$.</p>
	<p>Figure 5b. The viewing of the volume hologram formed as in figure 5a is shown.</p> <p>In this case the viewing wave is selected to have the same direction as the original reference beam used to form the hologram. $\beta_v = \alpha_R$.</p> <p>The trial image wave is assumed to leave in the direction from which the original object wave arrived. $\beta_{+1} = \alpha_O$.</p>
	<p>Figure 5b. The red-shift that occurs when a white light viewing beam is rotated closer to normal incidence is modelled.</p>

Underlying the discovery of the hypothetical fringe method is a slightly different interpretation of wave constructive and destructive wave interference that I have developed by emphasizing in my thinking the fact that holography works! To be more specific, constructive wave interference has the little noted characteristic of identifying regions of space where two equal amplitude and equal wavelength interfering plane waves are identical. The bright interference fringes formed by said plane waves mark planar regions throughout which the two wave disturbances are everywhere and always identical. Wave energy flows through these regions, the holographic film absorbs some of this energy, and, hence, holographic film automatically records locations in space where the two waves, object wave and reference wave are 'the same' It can be noted that the 'dark fringes' the locations of destructive interference mark the places where the two waves are 'most different.' In the dark regions where one wave has maximum positive disturbance, the other has maximum negative disturbance, and the two waves cancel. And, of course, in these regions, the holographic film remains clear.

So, again, the bright fringes, whose location is recorded in the hologram, mark the places where the reference beam and the object beam are the same, and Moiré diagrams very well represent this process, and permit related equations to be written. Then, as demonstrated above, the 'hypothetical fringes' that the viewing wave and the image wave would form should they be permitted to exist in the same region of space do correctly predict all permitted image wave directions for a thin transmission hologram. The values for β_{+1} and β_{+2} stated in the figure 4 captions agree can be shown to agree with predictions of the thin grating equation. In fact the thin grating equation can be derived using hypothetical fringe concept demonstrated in figure 4.

The author's 'short explanation' of why the method of hypothetical fringes works is that, the hypothetical fringes identify the locations where the viewing beam and the image wave are identical in phase, just as happens with the reference and object wave during hologram formation. And, in the simplest model, this is exactly the requirement that must be met at each and every holographic grating opening. This is so, because the viewing wave is the source of the image wave. Thinking of Huygens principle, each point on the viewing wave front that exists in a particular grating opening is the source of the image wave that travels outward from the illuminated hologram. In the grating openings, the image wave is the viewing wave, and vice versa. There is one and only one wave at any point in each grating opening. Having a hypothetical fringe pass through each grating opening guarantees that the 'sameness' requirement between the viewing wave and image wave is satisfied.

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